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The real Wick rotations in quantum gravity

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Abstract: We discuss Wick rotations in the context of gravity, with emphasis on a non-perturbative Wick rotation proposed in hep-th/0103186 mapping real lorentzian metrics to real euclidean metrics in proper-time coordinates. As an application, we demonstrate how this Wick rotation leads to a correct answer for a two dimensional non-perturbative path-integral.

Keywords: Models of Quantum Gravity, 2D Gravity.

Contents

1. Introduction

The evaluation of the path-integral for any field theory, essentially involves a rotation to euclidean signature space-times. This renders the complex measures and oscillatory weight factors $\mathcal{D}ge^{\imath S}$ real and the functional integral computable. One then Wick rotates back in the final answer to get lorentzian quantities, or invoke the Osterwalder-Schraeder theorems. The usual methods used for field theories in flat space-times are difficult to extend to gravity as there is no distinguished notion of time. There have been attempts to define a euclidean gravitational path-integral and relate them to lorentzian quantities. Initially complex weights were replaced by real Boltzmann weights and the sum performed over all possible Riemannian metrics [1]. This prescription is ad hoc as there is no a priori relation between the lorentzian and Riemannian sums. However, if one believes that lorentzian quantities are the relevant physical quantities, then a prescription must exist for identifying the corresponding euclidean metrics to be summed over. The usual Wick rotations pertain to static metric solutions of Einstein's equations. Given the time coordinate, t, Wick rotation amounts to going to the space of complex metrics, and integrating over the imaginary time axis. As evident, this prescription cannot be directly extended to a non-perturbative configuration space path integral where there is a priori no distinguished notion of time.

The solution to this problem was proposed in [2], where one gauge fixes the metric and performs a Wick rotation on the space of gauge fixed metrics. This 'non-perturbative' Wick rotation is a first step in mapping lorentzian path-integrals to euclidean path-integrals, in configuration space. The mapping crucially does not involve a complexification of space-time and maps the real lorentzian metrics to corresponding real Riemannian metrics in the path-integral. To verify that our proposal works, we discuss some examples where an explicit answer can be obtained. In particular we show how in 2 dimension, the only case where the 'non-perturbative' sector can be explicitly evaluated, a correct 2-loop answer is obtained. We show how the rotation relates the lorentzian and euclidean sectors of the theory. A completely independent evaluation of the euclidean path integral in 2 dimensions would include 'baby universes' and hence not satisfy the stringent causality requirements which the lorentzian histories have to satisfy. We first define the path-integral in lorentzian space, gauge fix the path-integral, Wick rotate to euclidean space to perform the computations. We also restrict the topology of the manifold to be fixed. The Faddeev-Popov determinant which arises due to gauge fixing gives the effective action in proper-time coordinates with the only degree of freedom being g_{11} component of the metric. Using a heuristic argument by coupling 2 dimensional gravity to matter, we show that this component of the metric has the product form $g_{11} = \gamma(t)\gamma(x)$. This immediately reduces the problem to that of quantum mechanics, and the zeta function regularisation of the Faddeev-Popov determinant gives the effective action as a quantum mechanical action. The path-integral can be explicitly evaluated once this is determined. We then rotate the final result to obtain the physical lorentzian answer. The answer obtained in the end is same as that obtained in a discretized evaluation of the path-integral [3]. A continuum calculation of the path-integral in proper-time coordinates had been previously obtained in [4]. Our approach is along the formalism of [2], where we derive the effective action from the Faddeev-Popov determinant arising in the measure unlike [4]. *Note the choice of gauge (the Proper-time gauge) plays a crucial role in all our arguments on the Wick Rotation as lorentzian concepts like causality are manifest, which restricts the path-integral.*

The following section is a detailed discussion of the Wick rotation proposed in [2], and the reason for it being non-perturbative is pointed out. The discussions in this section are similar as those in [5]. The third section enumerates the applications. An explicit calculation of the two dimensional path-integral is given, and there is a short review of the conformal mode cancellation of [2] to complete our argument. We end with a discussion.

2. The Non-perturbative 'Wick Rotation'

Since the *perturbative* path integral does not define a fundamental theory of quantum gravity, we have to consider, non-perturbative evaluations (see [6] for recent examples). As there is no distinguished notion of time in gravity, definition of euclidean path-integral is not well defined. A non-perturbative Wick rotation however can be implemented on the space of gauge fixed metrics as shown in [2]. Here, after fixing the gauge in the *lorentzian* path integral, the appropriate gauge-invariant metrics are written in coordinates with a distinguished time. We then 'Wick rotate' in the time-like component of the gauge invariant metric. To implement this, we need to identify a suitable gauge, and the

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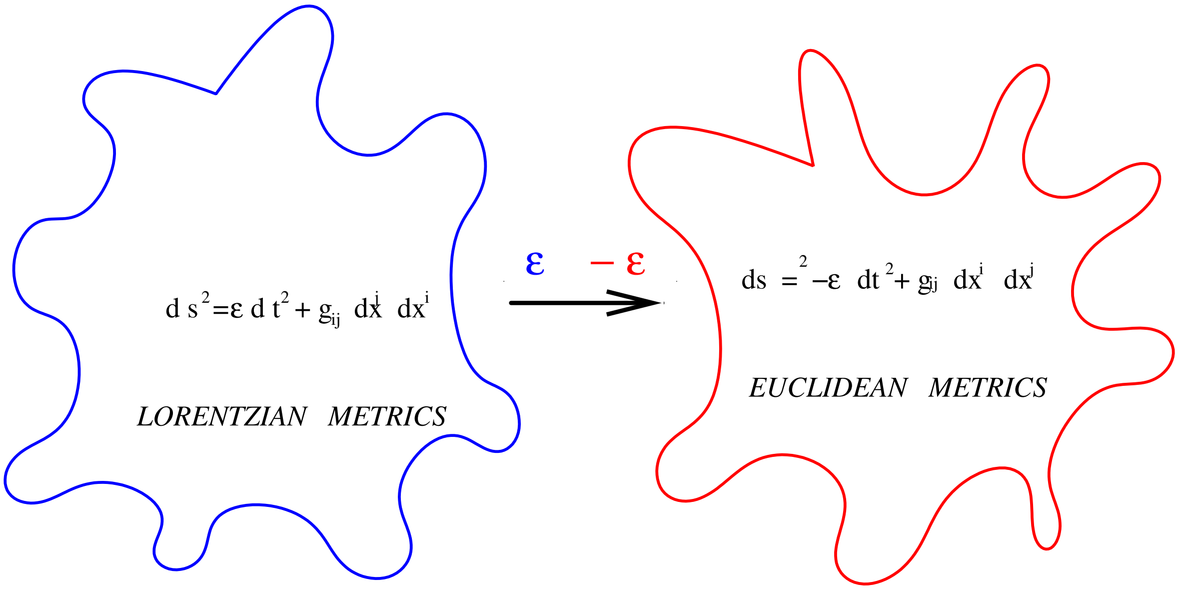

Figure 1: Mapping of gauge fixed metrics

proper-time gauge precisely fits in. Here the notion of time is distinguished, and causality is manifest. Our construction, including the 'Wick rotation', is non-perturbative and background-independent.

Our gauge is defined by the use of gaussian normal coordinates, where the d-dimensional metric takes the form

$$ds^2 = \epsilon \; dt^2 + g_{ij} dx^i dx^j \,, \quad \epsilon = \pm 1, \quad i = 1, 2, ..., d-1, \tag{2.1}$$

singling out a *proper time* t. The value $\epsilon = -1$ corresponds to lorentzian-signature metrics. Our generalized Wick rotation consists in mapping

$$\epsilon \mapsto -\epsilon, \tag{2.2}$$

while keeping g_{ij} *unchanged.* It has the properties summarized in figure 1:

- *Real* lorentzian metrics are mapped to *real* euclidean metrics, thus making the De-Witt metric and associated scalar products well-defined. Note that this is unlike the prescriptions used in [7] where complexification of the metric occurs. This is necessary for us as we need not only the action to be real but also the De-Witt metric $G^{\mu\nu\rho\sigma}$ in the configuration space of metrics, and the associated scalar products $\langle h, h \rangle = \int d^4x \sqrt{g} h_{\mu\nu} G^{\mu\nu\rho\sigma} h_{\rho\sigma}$ to be real too. A simple illustrative example is that of the de Sitter metric (foliated by flat hypersurfaces),

$$ds^2 = \epsilon \; dt^2 + e^{2\Lambda t} \; d\vec{x}^2 \,, \tag{2.3}$$

which is also a solution to the Einstein equations. The standard rotation $t \to it$ takes this to a complex-euclidean metric. Not only that, the scalar product defined above is clearly complex for d=4. The gaussian normalisation conditions used to define the correct measure in the form

$$\int \mathcal{D}h \exp^{-\langle h,h\rangle} = 1 \tag{2.4}$$

will no longer have a real exponent, and hence the 'euclideanised' integrals will also be ill defined. The Wick rotation to real Riemannian metrics which is achieved in (2.2) is a necessity here.

- The initial motivation for Wick rotation is of course the replacement of complex amplitudes in the path-integral by real weights. This is very well achieved in our prescription. The Complex amplitudes which appears in the path integral can be written in this gauge as $e^{i\sqrt{-\epsilon}S_\epsilon}|_{\epsilon=-1}$, with the action S as:

$$S_\epsilon[g] = -\frac{\epsilon}{16\pi G}\int d^dx\sqrt{\det g_{ij}}\left({}^{(d-1)}R - 2\Lambda - \frac{\epsilon}{4}G^{ijkl}_{(-2)}(\partial_0 g_{ij})(\partial_0 g_{kl})\right), \tag{2.5}$$

where $G^{ijkl} = (g^{ik}g^{jl} + g^{il}g^{jk} + Cg^{ij}g^{kl})/2$ is the DeWitt metric defined on the functional space of metrics and Λ the cosmological constant. (The one-parameter ambiguity C gets fixed by the form of the kinetic term in the action to $C = -2$.) Now clearly, the Wick rotation prescription given as $\epsilon \to -\epsilon$ helps in mapping the complex weights to real Boltzmann weights $e^{-S_{eu}}$.

- The map is one-to-one and onto the space of gauge-fixed euclidean metrics. We assume that metrics of the form (2.1) are in a suitable sense dense in the quotient of metrics modulo diffeomorphisms, see [2] for a detailed discussion.

- The non-perturbative nature of our Wick rotation becomes manifest when we look at *solutions* to Einstein's equations. In general, lorentzian solutions of the Einstein equations are not mapped to solutions of the Einstein equations in euclidean space by (2.2). In proper-time coordinates, as written in (2.1), the entire ϵ dependence can be absorbed into a rescaling of the time coordinate by $\sqrt{\epsilon}$ (See appendix A). Thus lorentzian and euclidean solutions ($g^L_{ij}(\tau,r) = g^E_{ij}(\tau,r)$) have the same functional dependence on the scaled parameter $\tau = \sqrt{\epsilon}t$, however, $\epsilon \to -\epsilon$ would implement the change $t \to \imath t$. Note that in our proposal for the Wick rotation, we assume that g_{ij} is independent of ϵ. Thus we are essentially not looking at Classical metrics, or Saddle points of the path-integral. The usual 'common-sense' associated with Einstein's equation solutions cannot be implemented in our space of metrics.

- Another positive point of this Wick rotation, is the fact that since $g_{ij}(t)$ remains unchanged, whatever properties the lorentzian metric has as a function of lorentzian time are retained in the euclidean case. As an example, the metric for desitter space (foliated by spheres, and $\Lambda = 1$) is:

$$ds^2 = -dt^2 + \cosh^2 t(d\chi^2 + \sin^2\chi d\Omega^2) \tag{2.6}$$

The standard $t \to it$ would give,

$$ds^2 = dt^2 + \cos^2 t(d\chi^2 + \sin^2\chi d\Omega^2) \tag{2.7}$$

which is pathological at $t = \pm\pi/2$.

However, our Wick rotation gives

$$ds^2 = dt^2 + \cosh^2 t(d\chi^2 + \sin^2 \chi d\Omega^2) \tag{2.8}$$

with the result that the euclidean metric is not pathological at $t = \pm\pi/2$. Hence, once we have fixed the configuration space of metrics for the lorentzian case, we can retain the mapped euclidean set of metrics in the sum.

The prescription (2.2) and the $t \to it$ agree only in a few cases, for example, the Minkowski and Schwarzschild metrics (which are static). If we were primarily interested in a perturbative treatment around given classical solutions, we might worry about the second last property. However, we need a Wick rotation that works on the space of *all* gauge fixed metrics, since this is the space we wish to sum over in the non-perturbative path integral, for given initial and final boundaries.

We conclude that in a non-perturbative context the prescription $\epsilon \mapsto -\epsilon$ is perfectly well-defined, and clearly preferred to a rotation to imaginary (proper) time.

3. Applications

The Wick rotation discussed in the previous section has to be implemented in a actual path integral to illustrate that it actually works. 2 dimensions is simplest to start with, and a full non-perturbative path-integral can be evaluated.

3.1 PI in 2 dim

The exact integral in 2 dimensions, helps in illustrating our Wick rotation, which is specific to the proper-time gauge we have chosen. We define the path-integral in two dimensional lorentzian geometry before 'Wick rotating' to euclidean space-times. The choice of our gauge helps us in realising lorentzian conditions manifestly e.g.

1. The existence of a global proper-time coordinate ensures that Topology is restricted (eg: formation of baby Universe would require a degenerate metric at the point of splitting, with the light cones pointing towards different time directions subsequently. See for reference [10, 11].)

2. Since propagator is evaluated between proper-time slices with the proper-time flowing from $[0, T]$, causality is manifest in this gauge.

The path-integral is

$$Z = \int \mathcal{D}g^{\mathrm{P}}_{\mu\nu} J \exp(-\imath S) \tag{3.1}$$

where J is the Faddeev-Popov determinant (FP) which arises due to the gauge fixing to a metric $g^{\mathrm{P}}_{\mu\nu}$. After determining the dependence of the FP on the metric variable (which in two dimensions has only one free component), one proceeds to evaluate the entire path-integral. (Here we have removed the diffeomorphism degrees of freedom.)

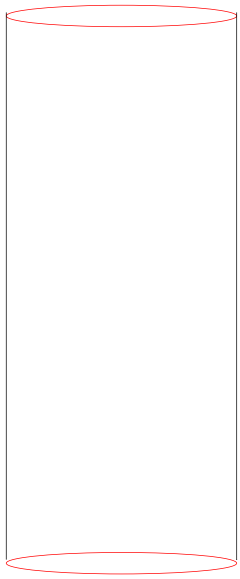

Figure 2: Cylinder topology

The gauge fixed metric $g^{\mathrm{P}}_{\mu\nu}(t,x)$ has the following form:

$$ds^2 = \epsilon dt^2 + \gamma(t,x)dx^2\,, \tag{3.2}$$

with $\gamma(t,x)$ being the g_{11} component of the metric. Before we start with evaluation of the P.I., we look at the classical phase space in proper-time coordinates. Given the two dimensional classical action $S = \int \sqrt{g}R + \Lambda \int \sqrt{g}$, ($\Lambda$ is the cosmological constant), one has to impose the constraint $T_{01} = 0$, to determine the classical configuration space(appendix B). Imposing this condition reduces the configuration space such that $\partial_t \partial_x \ln \sqrt{\gamma}(t,x) = 0$, and hence $\gamma = \gamma(t)\gamma(x)$. With the above variables, the problem reduces to one of quantum mechanics. This restriction, is also closely related to the restriction to cylinder topology. As seen in discrete calculations [3], one essentially studies correlations of loops $\int \sqrt{\gamma}dx = l(t)$ in time, with the space direction integrated out. (One can attribute this to lorentzian cobordism which restricts possible classical geometries between two given boundary data in 2 dimensions. For discussions on these issues see [11, 12]).

It is an interesting exsercise to determine the behaviour of 2-dim gravity coupled to matter fields in this gauge. Using a heuristic argument, we demonstrate how quantisation in this gauge is different from a ab initio quantisation in conformal gauge.

As explained in [9] the matter integral can be done exactly, and the effective energy momentum tensors determined. In other words: $\langle T_{\mu\nu}\rangle = \frac{\delta F}{\delta g^{\mu\nu}}$, with

$$Z^M = \int \mathcal{D}X \exp\left(-\frac{1}{2}\int \sqrt{g}g^{\mu\nu}\partial_\mu X \partial_\nu X\right) = \exp(-F) \tag{3.3}$$

The conformal anomaly (in euclidean space) is given as [9]

$$g^{\mu\nu}\langle T_{\mu\nu}\rangle = \left(\frac{D}{24\pi}\right)[R(\xi) + \text{constant}] \tag{3.4}$$

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The D denotes the number of scalar fields. Note, the presence of the conformal anomaly does not break the diffeomorphism invariance of the quantum effective action. Now, we impose the constraint in proper-time gauge such that $\langle T_{01}\rangle = 0$. This can be implemented in the effective action by using the following property:

$$g_{01} = h_{01} \tag{3.5}$$

such that

$$\left.\frac{\delta Z^M}{\delta h_{01}}\right|_{h_{01}=0} = \langle T_{01}\rangle = 0 \tag{3.6}$$

This restriction in proper-time gauge immediately implies that the correlations are important only across two proper-time slices. Let us try to see what this implies for conformal gauge. The transformations which take the conformal set of coordinates to the proper-time coordinates are as given below. The metric in conformal gauge can be written in terms of (ξ_0, ξ_1) coordinates as the following:

$$ds^2 = e^{2\phi(\xi)}\left(\epsilon d\xi_0^2 + d\xi_1^2\right) \tag{3.7}$$

Where ϕ denotes the conformal mode and the $\epsilon \to -\epsilon$ denotes a corresponding mapping to euclidean metrics in conformal gauge. This mapping is of course not equivalent to the mapping defined in the proper-time gauge. For simplicity at present we take the ϵ to be same. The coordinate transformations are very simple, and gives a relation between the derivatives of the coordinates as (see appendix B):

$$\epsilon\, \dot{t}^2 + t^{'2} = \epsilon\, e^{2\phi} \tag{3.8}$$

$$\epsilon \dot{x}^2 + x^{'2} = \frac{\exp(2\phi)}{\gamma} \tag{3.9}$$

$$\text{with}\dot{t} = \pm\sqrt{\gamma}x', \qquad t' = \pm\sqrt{\gamma}\dot{x} \tag{3.10}$$

(Where $\dot{t}$ denotes differentiation of t with respect to ξ_0 and x' denotes differentiation of x with respect to ξ_1). This coordinate transformation can be implemented on the energy momentum tensors in (3.4).

To quantise in conformal gauge, one imposes the constraint $\langle T_{01}^{CF}\rangle = 0$, and one can use the diffeomorphism transformation to see what it implies in the proper-time coordinates. Using the coordinate transformations, one finds:

$$\langle T_{01}\rangle^{CF} = \dot{x}\, x'\left(\epsilon\langle T_{00}\rangle - \frac{1}{\gamma}\langle T_{11}\rangle\right) = 0 \tag{3.11}$$

Where $\langle T_{01}\rangle = 0$ has been implemented. Clearly for the above to be correct, one is left with the following options:

$$\dot{x} = 0\,, \qquad \text{or} \qquad x' = 0 \qquad \text{or} \tag{3.12}$$

$$\epsilon\langle T_{00}\rangle = \frac{1}{\gamma}\langle T_{11}\rangle \tag{3.13}$$

The last condition is disallowed from equations which arise determining the energy momentum tensors due to (3.4) and the conservation equations $\nabla^\mu\langle T_{\mu\nu}\rangle = 0$ (appendix B).

It also happens that when we continue to lorentzian signature by (3.9),the condition $x' = 0$ is not possible. What we have left is $\dot{x} = 0$ and hence by (3.10), $t' = 0$, which implies again that $\gamma = \gamma(x)\gamma(t)$ and $\phi(\xi) = \phi(\xi_0)$. Note in principle we should have used the inverse transformations, i.e. transformed $\langle T_{01}\rangle = 0$ to conformal gauge, but conclusions would have been same.

An ab initio quantisation in conformal gauge would not have the reduced the conformal mode to be time dependent, and yielded the same result. *Thus our choice of gauge on which we can implement the lorentzian conditions in a manifest manner, is crucial.*

The path integral takes the following form in the proper-time gauge:

$$Z = \int \mathcal{D}\gamma J(\epsilon, \gamma) e^{i\sqrt{\epsilon}\Lambda \int d^2x \sqrt{\gamma}}. \tag{3.14}$$

The Wick rotation which takes $\epsilon \to -\epsilon$ is very simple , and we compute the entire path integral calculation in euclidean space. *Albeit hereafter the computations are similar to a euclidean P.I., one must realise that an ab initio calculation in Conformal gauge euclidean P.I. would not have resulted in the same answer, unless one implemented the lorentzian conditions and rotated the corresponding proper-time obtained as a function of the conformal mode.*

The main task is the evaluation of the Faddeev-Popov determinant J and it's dependence on γ. To evaluate the determinant explicitly, we refer to [2] and use the gaussian normalisation condition to fix the exact value. We expand the metric around the diffeomorphism orbits:

$$\delta g_{\mu\nu} = \delta g^{\mathrm{PT}}_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = \delta g^{\mathrm{PT}}_{\mu\nu} + (L\xi)_{\mu\nu} \tag{3.15}$$

For our purposes we set C=0, a value predetermined by the action we have chosen. (Note that this is very specific to 2-dimensions) (3.15) is then used in:

$$\int \mathcal{D}\delta g_{\mu\nu} \exp\left(-\langle \delta g, \delta g\rangle\right) = 1$$

Denoting the jacobian of the transformation as J, the induced measure on the tangent space is determined. Note that since the exponent is diffeomorphism invariant, we determine the scalar product at the base point on the gauge slice. As given in appendix C, the Faddeev-Popov determinant can be written as a product of three scalar determinants. This follows as after a few redefinitions as in appendix C, the entire exponent can be written as a sum of complete squares.

$$\int \mathcal{D}\delta g_{11} \mathcal{D}\xi_0 \mathcal{D}\xi_1 \exp\left[-\int d^2x \left(\delta g_{11}(\nabla_{11})\delta g_{11} + \xi_0'(\nabla_0)\xi_0' + \xi_1'(\nabla_1)\xi_1'\right)\right] \tag{3.16}$$

Where the vectors have been re-defined in order to complete the squares as given in appendix C. The gaussian integrals can now be done to yield the appropriate jacobians in tangent space. The jacobians evaluated here also determine the measure in base functional space. The Faddeev-Popov determinant is obtained as a product of three scalar determinants.

$$J^2 = \det{}_{\mathrm{S}}(\nabla_0)\det{}_{\mathrm{S}}(\nabla_1)\det{}_{\mathrm{S}}(\nabla_{11}) \tag{3.17}$$

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where the explicit expressions for the $\nabla_0, \nabla_1, \nabla_{11}$ are given as in appendix C. Being the simplest, we concentrate on ∇_0 first.

$$\det_S(\nabla_0) = \det_S \left[4 \left(\partial_0^\dagger \partial_0 + \frac{1}{2\gamma} \partial_1^\dagger \partial_1 + \frac{(\ln \dot{\gamma})^2}{4} \right) \right] \tag{3.18}$$

The operator in the above is thus:

$$P = \sqrt{\gamma} \left[4 \left(\partial_0^\dagger \partial_0 + \frac{1}{2\gamma} \partial_1^\dagger \partial_1 + \frac{(\ln \dot{\gamma})^2}{4} \right) \right] \delta(t - t')\delta(x - x') \tag{3.19}$$

The determinant of this operator then satisfies the following eigenvalue equation:

$$P\psi = n\sqrt{\gamma}\psi \tag{3.20}$$

where the eigenfunctions ψ with eigenvalues n, have to satisfy appropriate boundary conditions on $t = 0$ and $t = T$ surfaces. We impose Dirichlet boundary conditions, and as shown below, assume that the operator equation can be solved by a separation of variables. Once the separable solutions are found, any other solution can be expressed as a linear combination of them [14]. To achieve separability of the solution, we write the operator equation in the following form

$$4\frac{\gamma(t)}{\psi(t)} \left(\partial_0^\dagger \partial_0 + \frac{(\ln \dot{\gamma})^2}{4} \right) \psi(t) - n\gamma(t) = \frac{1}{2\psi(x)\gamma(x)} \left(\partial_1^\dagger \partial_1 \right) \psi(x)\,. \tag{3.21}$$

What is evident from the above is the fact that the above equation is easily separable in the variables but with the consequence that the eigenvalue n depends on the time dependent operators only. This is a clear indication of the reduction of the system to a quantum mechanical system with the reduced action depending only on the time coordinate. (The effective action is derived from $J = \Pi n$. Also the splitting we found in $\gamma = \gamma(t)\gamma(x)$ contributes crucially to our conclusion.) The eigenvalue equation for the t dependent part becomes now

$$4\gamma(t) \left(-(\partial_0 + \frac{1}{2}\ln \dot{\gamma})\partial_0 \right) \psi(t) + \gamma(t)(\ln \dot{\gamma})^2\psi(t) - n\gamma(t)\psi(t) = -\alpha\psi(t) \tag{3.22}$$

where α is a positive constant independent of t, x.

Finding the effective action hence reduces to the problem of finding the determinant of the operator

$$4(\partial_0^\dagger \partial_0) + (\ln \dot{\gamma})^2 + \frac{\alpha}{\gamma}\,. \tag{3.23}$$

In order to determine the above, we perform a zeta function regularisation. We follow [13] to perform this. Since it is difficult to evaluate the exact determinant, we try to guess the dependence on the $\gamma(t)$, by assuming that the effective action has the form

$$\Gamma = \int dt \left(f(\gamma)(\partial_0\gamma)^2 + V(\gamma) \right) \tag{3.24}$$

with $f(\gamma)$ an arbitrary function. Thus, for configurations which have $\dot{\ln\gamma} = 0$, we recover $V(\gamma_0)$, where $V(\gamma_0)$ is the potential depending on the constant value γ_0. In other words the potential is simply given by the zeta function regularisation of the operator in (3.23) with the value of γ_0:

$$\int dt V(\gamma_0) = -\zeta'(0) \tag{3.25}$$

Note that the metric in (3.2) is still non-constant due to the dependence on $\gamma(x)$.

Thus, we consider the operator for γ_0 which reduces to the following form

$$-4\partial_0^2 + \frac{\alpha}{\gamma_0} \tag{3.26}$$

The Heat Kernel for this operator satisfies the following equation:

$$-\left(4\partial_0^2 - \frac{\alpha}{\gamma_0}\right) G(t, t', \tau) = -\partial_\tau G \tag{3.27}$$

with the boundary condition that

$$G(t, t', \tau = 0) = \frac{1}{\sqrt{\gamma}_0}\delta(t - t') \tag{3.28}$$

(the metric in this direction is flat) The heat Kernel $G(t, t', \tau)$ can be solved to give:

$$G(t, t', \tau) = \sqrt{\frac{\pi}{4\tau\gamma_0}} e^{-\frac{(t-t')^2}{16\tau} - \frac{\alpha}{\gamma_0}\tau} \tag{3.29}$$

The ζ function is thus:

$$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty d\tau \tau^{s-1} \int dt \sqrt{\frac{\pi}{4\tau}} e^{-\frac{\alpha}{\gamma_0}\tau} \tag{3.30}$$

The $\int dt$ is not infinite, and cancels with the volume element in the l.h.s. of (3.25). The task thus reduces to the determination of

$$\zeta'(0) = \frac{\sqrt{\pi}}{2} \frac{\Gamma(s - 1/2)}{\Gamma(s)} \left(\frac{\gamma_0}{\alpha}\right)^{s-1/2} \left[\ln\left(\frac{\gamma_0}{\alpha}\right) + \Psi\left(s - \frac{1}{2}\right) - \Psi(s)\right]\Bigg|_{s=0} \tag{3.31}$$

At this juncture, we keep only finite terms as $s \to 0$, and hence only the term proportional to $\Psi(s)/\Gamma(s)$ of the above expression. It is easily justifiable as the rest of the terms, due to the pole of $1/\Gamma(s)$, at $s \to 0$ contribute vanishingly to $e^{-\zeta'(0)}$.

The potential has a dependence of the form:

$$\int dt V(\gamma_0) \approx \int dt \sqrt{\gamma_0} \frac{1}{\gamma_0} \tag{3.32}$$

Next we look at the operator ∇_1, which has the form as given in the appendix:

$$\nabla_1 = \frac{4}{\gamma^2} \nabla_1^0 \left(1 + \left(\frac{\nabla_1^0}{\gamma}\right)^{-1} \nabla_1^1\right) \tag{3.33}$$

with

$$\nabla_1^0 = \left(\partial_1^\dagger \partial_1 + \frac{\gamma}{2} \partial_0^\dagger \partial_0 + \frac{\gamma}{2} (\ln \dot{\gamma}) \partial_0 + \frac{(\ln \gamma')^2}{4} \right) \tag{3.34}$$

The operator in (3.34) can be cast in a form similar to the operator ∇_0 with a measure of $1/\sqrt{\gamma}_0$. However, this does not modify the final answer, and one gets a very similar analyses as above, except for a change of factor 2. And the contribution to the effective potential is of the form $\int dt 1/\sqrt{\gamma_0}$. The rest of the factorised operator in (3.33) does not contribute to the potential (see appendix C). The full form of the potential *is* determined above, the contributions due to nonzero $\ln \dot{\gamma}$, will contribute to the derivative terms in the action. Though one may think that a constant γ can be absorbed into a rescaling of the coordinate, note that the conformal anomaly precisely prevents this. To determine the derivative term in (3.24), we resort to the demand of locality of the action as previous work in other gauges [9] have local actions. To see what such a term in the action should look like we make a scaling of just the t coordinate to go to conformal gauge with the metric

$$ds^2 = \gamma(dt'^2 + dx^2) .$$

The local term in these set of coordinates would be $\int dt' \frac{1}{4} (\partial_t' \ln \sqrt{\gamma})^2$. Translating this back to original coordinates we conclude that the effective action can be written as:

$$I = \int dt \left(\frac{1}{4\sqrt{\gamma(t)}} \left(\frac{d\sqrt{\gamma}(t)}{dt} \right)^2 + \frac{\beta}{\sqrt{\gamma}(t)} + \Lambda \sqrt{\gamma}(t) \right) \tag{3.35}$$

The constant β is finite. Remarkably, the above coincides with the effective action found in [4]. To extract the finite part of our effective action, the observation of separability proved crucial. In principle we have not argued for the absence of higher derivative terms, however conditions of locality are stringent. A full determination of the determinant for nonzero $\ln \dot{\gamma}$ should ultimately give (3.35), and shall be interesting to evaluate.

Once the action has been determined, one needs to complete the evaluation of the propagator

$$G(l_1, l_2, T) = \int_{l_1}^{l_2} \frac{\mathcal{D}l}{l} \exp(-I(l)) \tag{3.36}$$

where we are now using the variable $\sqrt{\gamma(t)} = l$. Though the action is quantum mechanical, it is not clear that the evaluation of (3.36) is simple. An exact evaluation is left for a future publication. Instead, one resorts to the following representation of the propagator.

$$G(l_1, l_2, T) = \langle l_1 | e^{-HT} | l_2 \rangle \tag{3.37}$$

Where H is the hamiltonian corresponding to (3.35). This is similar to the approach in [4], and one refers the readers to the calculations there, and quotes the final result.

$$G(l_1, l_2, T) = \sqrt{\Lambda} e^{-\sqrt{\Lambda}(l_1 + l_2) \coth(\sqrt{\Lambda} T)} \frac{1}{\sinh(\sqrt{\Lambda} T)} I_a \left(\frac{2\sqrt{\Lambda l_1 l_2}}{\sinh(\sqrt{\Lambda} T)} \right) \tag{3.38}$$

This amplitude is the same obtained in different approaches in [3, 4], with $a = 1$ coincides with the result found in [3]. However, the interesting part is the rotation back in the final result to get a physical lorentzian answer. The Wick rotation initially involved setting

$\epsilon \to -\epsilon = 1$ to get the euclidean set of metrics to be summed over. In the final answer, this rotation is hidden in the definition of proper-time T, or the euclidean time is $\sqrt{\epsilon}T$. (This can be traced back to equation (3.14), and the fact that γ is independent of ϵ). Substituting $T \to iT$, a physical lorentzian answer is obtained.

In two dimensions, we implemented all the properties of Wick rotation stated in section 2, and succeeded in getting an exact and correct answer for the non-perturbative path-integral. Hence it is interesting to see how far we can implement this in higher dimensions.

3.2 In higher dimensions

Our formalism crucially rests on the proper-time gauge. In higher dimensions, as discussed in [2], the action is as given in (2.5).

The euclidean and lorentzian actions are related by $\epsilon \mapsto -\epsilon$. We will set $\epsilon = 1$ in what follows.

As discussed in [2], we showed that the above helps in the identification of the negative definite part of the action, which is real unlike what a complexification of the lorentzian metric would achieve.

$$S^{\mathrm{ND}} = -\frac{(d-1)(d-2)}{16\pi G} \int d^d x \; \mathrm{e}^{(d-1)\tilde{\lambda}} (\partial_0 \tilde{\lambda})^2 \tag{3.39}$$

with

$$\tilde{\lambda} = \lambda + \frac{\log \bar{g}}{2(d-1)} \,. \tag{3.40}$$

(λ is the conformal mode $g_{ij} = e^{2\lambda}\bar{g}_{ij}$). The full action can be written as $S = S^{\mathrm{ND}}(\tilde{\lambda}) + S^{\mathrm{rest}}(\bar{g}_{ij}, \tilde{\lambda})$, where we refrain from giving the remainder S^{rest} explicitly [2].

The main task is to now deal with the presence of the e^{λ} dependence in the measure for λ. In other words we need a transformation which is similar to that used in 2-dimensional gravity [16]:

$$\mathcal{D}\tilde{g}_{e^{\lambda}\bar{g}} \mathcal{D}\tilde{\lambda}_{e^{\lambda}\bar{g}} \to J(\lambda, \bar{g}) \mathcal{D}\tilde{g}_{\bar{g}} \mathcal{D}\tilde{\lambda}\bar{g} \tag{3.41}$$

We assume that this is achieved with the jacobian J, a local function of the fields and is of the form e^{-S_L}. For obvious symmetry reasons the local action is of the following form

$$S_L = \int d^d x \sqrt{\bar{g}} \left[A \; \left(\partial_0 \tilde{\lambda}\right)^2 + B \; \bar{R} + C \; (\bar{R}_{ijkl}\bar{R}^{ijkl}) + D \; (\nabla^2 \bar{R}) + \cdots \right] \tag{3.42}$$

where A, B, C, D,... are constants whose value will depend on the regularisation chosen. What is however clear that the kinetic term for the conformal mode appears in the same form as in the original action and can be absorbed back into the action with just a renormalisation of the coupling constant Thus inspired by a conjecture lying at the heart of 2d Liouville quantum gravity [16], we now assume that an elimination of the conformal factors from the space-time measures in the exponent and the determinants (such that

$\sqrt{g}e^{(d-1)\lambda} \mapsto \sqrt{g}$) is equivalent to a renormalization of the action. We now pull all jacobians and other $\tilde{\lambda}$-independent terms out of the $\tilde{\lambda}$-integral. The leading divergence of what is left under the integral is of the form

$$\int \mathcal{D}\tilde{\lambda}\ e^{\int d^d x \tilde{\lambda}\partial_0^\dagger \partial_0 \tilde{\lambda}}. \tag{3.43}$$

This formally yields $1/\sqrt{\det_S(-\partial_0^\dagger\partial_0)}$, which is badly divergent, since $\partial_0^\dagger\partial_0$ is a positive operator. Nevertheless, it is of exactly the same functional form as the inverse of scalar part of the Faddeev-Popov determinant as determined in [2]. One can then cancel the two divergent determinants, leaving behind the integral in $\tilde{g}_{ij}$. This completes the argument for the cancellation of the conformal divergence.

4. Discussions

In this article we argued for a novel type of Wick rotation which seems particularly suited for a *non-perturbative* treatment of the gravitational path integral. It is an example of how the need for going beyond perturbative treatments in quantum gravity forces us to abandon a well-loved, but essentially perturbative concept like imaginary time. As an application, we gave a 2dimensional non-perturbative path integral calculation and outlined how in higher dimensions, the conformal sickness of the Wick-rotated path integral is cured non-perturbatively in this new framework. The two dimensional path-integral revealed that the choice of proper-time gauge was crucial in our arguments as the lorentzian restrictions can be imposed manifestly here. The final result confirmed that the Wick rotation on the space of proper-time metrics does work in a non-perturbative set up. It shall be interesting to see how the lorentzian conditions obtained here translate to other gauges like the light cone gauge, and this answer reproduced from quantisations used in [17]. And in higher dimensions the cancellation of the conformal mode divergence leads to the hope that the path-integral can be evaluated to yield exact results.

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A. Proper-time gauge

The non-zero Christoffels:

$$\Gamma^0_{ij} = -\frac{\epsilon}{2} g_{ij,0} \tag{A.1}$$

$$\Gamma^i_{0j} = \frac{1}{2} g^{ik} g_{kj,0} \tag{A.2}$$

$$\Gamma^i_{lj} = \frac{1}{2} g^{ik}\left(g_{lk,j} + g_{jk,l} - g_{lj,k}\right) = \Gamma^{(2)\ i}_{\ \ lj} \tag{A.3}$$

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The Ricci Tensor components

$$R_{00} = \frac{1}{2}\left(g^{ik}g_{ik,0}\right)_{,0} + \frac{1}{4}g^{jm}g^{kn}g_{jn,0}g_{km,0} \tag{A.4}$$

$$R_{i0} = \Gamma^{(2)j}_{ij,0} - \frac{1}{2}\left(g^{jk}g_{ik,0}\right),j + \Gamma^{(2)k}_{ij}\frac{1}{2}g^{jm}g_{km,0} - \Gamma^{(2)j}_{jk}\frac{1}{2}g^{kl}g_{il,0} \tag{A.5}$$

$$R_{ij} = R^{(2)}_{ij} + \epsilon\left(g_{ij,0,0} + g_{ij,0}g^{lm}g_{lm,0} - g_{ik,0}g_{jm,0}g^{km}\right) \tag{A.6}$$

Scaling the time coordinate by $\sqrt{\epsilon}$, and putting the above in Einstein's equation, the entire ϵ dependence is removed from them (we also need to use $\epsilon^2 = 1$). So, one obtains $g_{ij} = g_{ij}(\sqrt{\epsilon}t, x)$

B. 2 dim and proper-time gauge

Though the 2 dimensional classical action is a total derivative, one knows that dynamics can be induced in the quantum corrected effective action due to the conformal anamoly. In anticipation of that, one can try and see what the classical reduced space can be, by looking at the constraint $T_{01} = 0$. This can be imposed by using

$$\left.\frac{\delta S}{\delta g^{01}}\right|_{g_{01}=0} = 0 \tag{B.1}$$

with $S = \int \sqrt{g}R + \Lambda\sqrt{g}$. Thus using the non-zero Christoffels, one finds

$$S = \int \sqrt{\gamma}\left[\epsilon R_{00} + \frac{1}{\gamma}R_{11}\right] + O(g_{01}^2) \tag{B.2}$$

With (quoting only the terms proportional to g_{01}) and $\epsilon = 1$,

$$R_{00} = \partial_x\left(\frac{1}{\gamma}\partial_t g_{10}\right) + \frac{1}{2\gamma^2}\partial_t g_{10}\partial_x\gamma \tag{B.3}$$

$$R_{11} = \partial_x\partial_t g_{01} - \frac{1}{4\gamma^2}g_{01}\partial_t\gamma\partial_x\gamma - \frac{1}{2\gamma}\left(\partial_t\left(\frac{g_{01}}{\gamma}\right)\partial_x\gamma - \partial_t\gamma\partial_x\left(\frac{g_{01}}{\gamma}\right)\right) \tag{B.4}$$

Using the above in (B.1), one finds that

$$T_{01} = \partial_x\partial_t \ln\sqrt{\gamma} = 0 \tag{B.5}$$

is solved by $\gamma = \gamma(t)\gamma(x)$

Thus classical configuration space is reduced by the above constraint. We can try to see what this implies for the conformal gauge: The coordinate transformations relating metric components in conformal gauge (3.7) and proper time gauge(3.2).

$$\epsilon\,\dot{t}^2 + \gamma\,\dot{x}^2 = \epsilon\,e^{2\phi} \tag{B.6}$$

$$\epsilon\,\dot{t}t' + \gamma\,\dot{x}x' = 0 \tag{B.7}$$

$$\epsilon\,t'^2 + \gamma x'^2 = e^{2\phi} \tag{B.8}$$

Dividing (B.6) by (B.8) and using (B.7) one obtains:

$$\dot{t} = \sqrt{\gamma}x' \tag{B.9}$$
$$t' = \sqrt{\gamma}\dot{x} \tag{B.10}$$

Putting the above two in (B.6) and (B.8) one obtains (3.8), (3.9).

To follow on the heuristic arguement, and Equation (3.13), we show why the condition $\langle T_{00}\rangle \neq \frac{1}{\gamma}\langle T_{11}\rangle$. The conformal anamoly equation, and the conservation equations $\nabla^\mu\langle T_{\mu\nu}\rangle = 0$ can be used to solve for the energy momentum tensors. For clarity we denote the the conformal anamoly in this gauge as $\langle T_{00}\rangle + \frac{1}{\gamma}\langle T_{11}\rangle = F(\gamma)$, where $F(\gamma)$ is an arbitrary function of the metric, and is usually proportional to the scalar curvature. In the next few lines $\langle T_{\mu\nu}\rangle \equiv T_{\mu\nu}$. The T_{01} component of the energy momentum tensor can be eliminated, yielding the following two equations:

$$\partial_1\left\{\frac{1}{\sqrt{\gamma}}\partial_1\left(\frac{1}{\sqrt{\gamma}}T_{11}\right)\right\}+\partial_0\left\{\sqrt{\gamma}\partial_0\left(\frac{T_{11}}{\sqrt{\gamma}}\right)\right\}+\partial_0\left\{\dot{\overline{\ln\gamma}}T_{11}\right\} = \partial_0\left\{\sqrt{\gamma}\partial_0(\sqrt{\gamma}F(\gamma))\right\} \tag{B.11}$$

$$\partial_0\left\{\sqrt{\gamma}\partial_0\left(\sqrt{\gamma}T_{00}\right)\right\} + \partial_1\left\{\frac{1}{\sqrt{\gamma}}\partial_1\left(\sqrt{\gamma}T_{00}\right)\right\} = -\partial_1\left\{\frac{1}{\sqrt{\gamma}}\partial_1\left(\sqrt{\gamma}F(\gamma)\right)\right\} - \partial_0\left\{\gamma\dot{\overline{\ln\gamma}}F(\gamma)\right\}. \tag{B.12}$$

The above two equations can be solved in principle to obtain a solution for the energy momentum tensor components, however it is obvious that $T_{00} = \frac{1}{\gamma}T_{11}$ cannot be a solution of the above. Hence Equation (3.13) are disallowed.

C. Faddeev-Popov determinant

This is a summary of the Faddeev-Popov determinant calculations in our gauge. An ab initio derivation of the determinant is required as our gauge is *non-covariant* and two dimensions is the easiest place to demonstrate that.

By ab-initio we mean that we start with the gaussian-Normalisation condition:

$$\int \mathcal{D}\delta g_{\mu\nu} e^{-\ \langle\delta g,\delta g\rangle} = 1 \tag{C.1}$$

where $\delta g_{\mu\nu}$ is fluctuation in tangent space around a given base point in the functional space of metrics.

We assume that our gauge fixed proper-time metric, is of the following form:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & \gamma \end{pmatrix} \tag{C.2}$$

with the following non-zero Christoffel symbols.

$$\Gamma^0_{11} = -\frac{1}{2}\dot{\gamma},\quad \Gamma^1_{01} = \frac{1}{2}(\dot{\overline{\ln\gamma}}),\quad \Gamma^1_{11} = \frac{1}{2}(\ln\gamma)'$$

$\dot{\gamma} \equiv \partial_t\gamma,\ \gamma' \equiv \partial_x\gamma$ denote the derivatives and $\gamma = \gamma(t)\gamma(x)$.

Using the above:

$$
\begin{aligned}
\langle \delta g, \delta g\rangle &= \int d^2x\sqrt{\gamma}\left[(2\nabla_0\xi_0)^2 + \frac{2}{\gamma}(\nabla_0\xi_1 + \nabla_1\xi_0)^2 + \frac{1}{\gamma^2}(\delta g_{11} + 2\nabla_1\xi_1)^2\right] \\
&= \int d^2x\sqrt{\gamma}\Bigg[\frac{(\delta g_{11})^2}{\gamma^2} + 4\dot{\xi}_0^2 + \\
&\qquad + \frac{4}{\gamma^2}\Bigg(\left\{(\xi_1') + \frac{1}{2}\dot{\gamma}\;\xi_0 - \frac{1}{2}(\ln\gamma)'\;\xi_1\right\}^2 + \\
&\qquad\qquad + \left\{\xi_1' + \frac{1}{2}\dot{\gamma}\;\xi_0 - \frac{1}{2}(\ln\gamma)'\xi_1\right\}\delta g_{11}\Bigg) \\
&\qquad + \frac{2}{\gamma}\left(\dot{\xi}_1 + \xi_0' - (\ln\dot{\gamma})\xi_1\right)^2\Bigg]
\end{aligned}
\tag{C.3}
$$

Using partial integrations, the above can be written in a more convenient form:

$$
\begin{aligned}
= \int d^2x\sqrt{\gamma}\Bigg[&\frac{1}{\gamma^2}\delta g_{11}^2 + 4\xi_0\left(\partial_0^\dagger\partial_0 + \frac{1}{2\gamma}\partial_1^\dagger\partial_1 + \frac{(\ln\dot{\gamma})^2}{4}\right)\xi_0 + \\
&+ \frac{4}{\gamma}\left\{\xi_0(\ln\dot{\gamma})(\partial_1 - \frac{1}{2}\ln\gamma')\xi_1\right\} - \frac{4}{\gamma}\left\{\xi_1\left(\partial_0 + \frac{1}{2}(\ln\dot{\gamma})\right)\partial_1\xi_0\right\} + \\
&+ \frac{4}{\gamma^2}\left\{\xi_1\left(\partial_1^\dagger\partial_1 + \frac{\gamma}{2}\partial_0^\dagger\partial_0 + \frac{\gamma\ln\dot{\gamma}}{2}\partial_0 + \frac{(\ln\gamma')^2}{4}\right)\xi_1\right\} + \\
&+ \frac{4}{\gamma^2}\xi_1\partial_1^\dagger\delta g_{11} + \frac{2}{\gamma}(\ln\dot{\gamma})\xi_0\delta g_{11}\Bigg]
\end{aligned}
\tag{C.4}
$$

(where $-\partial_0^\dagger = \partial_0 + 1/2(\ln\dot{\gamma})$, $-\partial_1^\dagger = \partial_1 - 1/2(\ln\gamma)'$)

One can then complete the squares to do the ξ integrations and obtain:

$$
= \int d^2x\frac{1}{\gamma^2}\delta g_{11}^2 + \xi_0'\nabla_0\xi_0' + \xi_1'\nabla_1\xi_1' - W_0 - W_1 \tag{C.5}
$$

where,

$$
\nabla_0 = 4\left(\partial_0^\dagger\partial_0 + \frac{1}{2\gamma}\partial_1^\dagger\partial_1 + \frac{(\ln\dot{\gamma})^2}{4}\right) \tag{C.6}
$$

$$
\begin{aligned}
\nabla_1 &= \frac{4}{\gamma^2}\left(\partial_1^\dagger\partial_1 + \frac{\gamma}{2}\partial_0^\dagger\partial_0 + \frac{\gamma}{2}(\ln\dot{\gamma})\partial_0 + \frac{(\ln\gamma')^2}{4}\right) + \\
&\quad + \frac{1}{\gamma}(\partial_0^\dagger - \ln\dot{\gamma})\partial_1(\nabla_0)^{-1}\frac{1}{\gamma}\partial_1^\dagger\left(\partial_0 - 2\ln\dot{\gamma}\right)
\end{aligned}
\tag{C.7}
$$

$$
W_0 = \frac{1}{\gamma}\delta g_{11}\ln\dot{\gamma}\nabla_0^{-1}\frac{\ln\dot{\gamma}}{\gamma}\delta g_{11} \tag{C.8}
$$

$$
\begin{aligned}
W_1 &= \delta g_{11}\frac{1}{\gamma}\left(\partial_1 + \ln\dot{\gamma}\partial_1^\dagger(\partial_0 - 2\ln\dot{\gamma})\partial_1\nabla_0^{-1}\right)\nabla_1^{-1}\times \\
&\quad\times\left((\partial_0^\dagger - \ln\dot{\gamma})\partial_1\nabla_0^{-1}\frac{\ln\dot{\gamma}}{\gamma} + \partial_1^\dagger\right)\delta g_{11}
\end{aligned}
\tag{C.9}
$$

After this, one can just evaluate the gaussian integrals in δg_{11}, ξ_0, ξ_1 in the (C.1) to get the jacobian in terms of three scalar determinants, $\det_S(\nabla_0)\det_S(\nabla_1)\det_S(\nabla_{11})$, with $\delta g_{11}(\nabla_{11} - 1/\gamma^2)\delta g_{11} = -W_0 - W_1$. For non-zero γ it becomes a Herculian task to determine the exact form of the effective action. Following the zeta function regularisation methods used in [13], the form of the effective action can be guessed by examining around a value of the metric for which $\ln\dot{\gamma} = 0$. This gives precisely the same action found in [4] for the determinant ∇_0 as explained in section 3.1. For the rest of the determinants of operators (C.7) and (C.9), we also do a zeta function regularisation. But most give diverging results due to presence of inverse of operators, and we absorb these in an infinite renormalisation of the effective action.

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